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Evaporation of a Kerr black hole by emission of scalar and higher spin particles

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Abstract

We study the evolution of an evaporating rotating black hole, described by the Kerr metric, which is emitting either solely massless scalar particles or a mixture of massless scalar and nonzero spin particles. Allowing the hole to radiate scalar particles increases the mass loss rate and decreases the angular momentum loss rate relative to a black hole which is radiating nonzero spin particles. The presence of scalar radiation can cause the evaporating hole to asymptotically approach a state which is described by a nonzero value of $a_* \equiv a/M$. This is contrary to the conventional view of black hole evaporation, wherein all black holes spin down more rapidly than they lose mass. A hole emitting solely scalar radiation will approach a final asymptotic state described by $a_* \simeq 0.555$. A black hole that is emitting scalar particles and a canonical set of nonzero spin particles (3 species of neutrinos, a single photon species, and a single graviton species) will asymptotically approach a nonzero value of a_* only if there are at least 32 massless scalar fields. We also calculate the lifetime of a primordial black hole that formed with a value of the rotation parameter a_*, the minimum initial mass of a primordial black hole that is seen today with a rotation parameter a_*, and the entropy of a black

hole that is emitting scalar or higher spin particles.

I. INTRODUCTION

The evolution of evaporating black holes is a process that has been studied in great detail. Recently we found that a black hole initially described by the Kerr metric which is radiating only massless scalar particles via the Hawking process, asymptotically evolves towards a state described by a rotation parameter $a \simeq 0.555M$ [1]. This is contrary to the conventional view of an evaporating black hole's evolution which is that an initially rotating black hole will spin down, evolving towards a Schwarzschild state before most of its mass has been lost. For black holes emitting solely scalar radiation, not only do initially rapidly rotating holes fail to spin down to a Schwarzschild state, but holes with initial values $a < 0.555M$ will actually spin up as they evaporate, again tending towards an asymptotic state with $a \simeq 0.555M$. Such a hole is losing angular momentum as it evolves, but is losing mass at a higher rate.

The evolution of evaporating black holes described by the Schwarzschild and Kerr metrics was studied in detail by Page [2,3]. In particular, Page numerically investigated the evolution of a rotating uncharged hole described by the Kerr metric that was emitting radiation via the Hawking process [3]. He found that a black hole emitting nonzero spin particles will lose angular momentum at a greater rate than it loses mass. Such a black hole will reach a state described by $a_* \equiv a/M \approx 0$ by the time it has lost approximately half its mass, allowing the hole to be described by the simpler Schwarzschild metric in the late stages of its evolution. While Page only investigated radiation by nonscalar fields in detail, he suggested that a hole that is emitting massless scalar particles might evolve differently. Page found that as the black hole evaporated the dominant mode for the nonscalar fields' radiation was that which had $l = m = s$, where l and m are the usual spherical harmonic indices and s is the spin of the field. Only a scalar field can radiate in the $l = 0$ mode, and, by Page's argument, one might expect this mode to be dominant for scalar radiation. This mode carries off energy from the hole, but no angular momentum. If a black hole is emitting scalar field radiation and the $l = 0$ mode for that radiation dominates the overall emission from both scalar and nonscalar

fields, then the hole could lose mass faster than it loses angular momentum. It would then evolve asymptotically to a state described by a nonzero value of a_*. This asymptotic value will hereafter be denoted by a_{*0}. This would mean that the hole would always be described by the Kerr metric rather than evolving asymptotically toward a Schwarzschild black hole. We have shown that this is the case if the black hole emits only scalar field radiation [1].

In this paper we discuss in more detail our results reported in Ref. [1]. We also determine whether or not the emission of massless scalar particles in addition to radiation from a canonical set of nonzero spin fields (3 separate neutrino, photon, graviton) via Hawking radiation can allow a black hole described by the Kerr metric to evolve towards an asymptotic state described by a nonzero value of a_*. We find that a single massless scalar field in addition to the canonical set of nonzero spin fields makes very little change in the evolution found by Page [3] and that the hole will still approach a state described by $a_* = 0$. In order to reach an asymptotic state described by a_{*0} we find one must allow the hole to emit radiation from a minimum of 32 massless scalar fields in addition to the canonical nonzero spin fields.

Our results combined with those of Page's results from Ref. [3] allow one to calculate the evolution of a Kerr black hole which is evaporating with an arbitrary collection of scalar and nonzero spin fields. The results of our evolutionary calculations from both purely scalar radiation and scalar plus the canonical set of nonzero spin fields are applied to calculate a number of quantities of interest. These include the lifetime of a Kerr black hole for a variety of initial conditions, the minimum initial mass a primordial Kerr black hole could have possessed assuming it was observed today with a rotation parameter of a_*, the initial mass of a primordial Kerr black hole which would have just disappeared today, and the evolution of the area, or equivalently the entropy, of such holes as they evolve.

In section II we discuss the mathematical formulae that will be used to determine the evaporating black hole's evolution. In Section III the numerical methods that were used are outlined. The results of our work are presented in Section IV, and a final summary offered in Section V. Our notation follows that of Page [3] and Teukolsky and Press [4]. We use natural units, such that $G = c = \hbar = k_B = 1$, except where otherwise specified.

II. MATHEMATICAL FORMULAE

We will assume that the black hole is in isolation and that enough time has passed so that the the black hole has lost all of its initial charge (if any), but is rotating and will therefore be described by the Kerr metric. The wave equation for the massless scalar field, $\Box\phi = 0$, separates in Kerr-ingoing coordinates [4] by choosing $\phi = R(r)S(\theta)e^{-i\omega\nu}e^{im\tilde{\phi}}$, where the angular function $S(\theta)$ is a spheroidal harmonic [5]. The radial function $R(r)$ satisfies

$$(\partial_r \Delta \partial_r - 2iK\partial_r - 2i\omega r - \lambda)\, R(r) = 0 \,, \tag{1}$$

where $\Delta = r^2 - 2Mr + a^2$, $K = (r^2 + a^2)\omega - am$, $\lambda = E_{lm\omega} - 2am\omega + a^2\omega^2$ and $E_{lm\omega}$ is the separation constant. A general solution to Eq. (1), expressed in terms of known functions, is not known, but asymptotic solutions can be found [4],

$$R \longrightarrow \begin{cases} Z_{\text{hole}} & r \rightarrow r_+ \\ Z_{\text{in}} r^{-1} + Z_{\text{out}} r^{-1} e^{2i\omega r} & r \rightarrow \infty \end{cases} . \tag{2}$$

The subscript "in" refers to an ingoing wave originating from past null infinity, "out" refers to an outgoing wave reflected from the hole that propagates toward future null infinity, and "hole" refers to the component of the wave that is transmitted into the black hole through the horizon at $r = r_+$. The amplification, Z (the fractional gain of energy in a scattered wave), is

$$Z = \left| \frac{Z_{\text{out}}}{Z_{\text{in}}} \right|^2 - 1. \tag{3}$$

Following Page [2] we express the rates at which the mass and angular momentum decrease by the quantities $f \equiv -M^2 dM/dt$ and $g \equiv -M a_*^{-1} dJ/dt$ respectively, which have been scaled to remove overall dependence on the size (mass) of the black hole. The coordinate t is the usual Boyer-Lindquist time coordinate. These quantities will determine the evolution of the black hole. They are defined by

$$\begin{pmatrix} f \\ g \end{pmatrix} = -\sum_{l,m} \frac{1}{2\pi} \int_0^\infty dx \frac{Z}{e^{2\pi k/\kappa} - 1} \begin{pmatrix} x \\ m a_*^{-1} \end{pmatrix} \,, \tag{4}$$

where $k = \omega - m\Omega$, $\Omega = a_*/2r_+$ is the surface angular frequency, $\kappa = \sqrt{(1-a_*^2)}/2r_+$ is the surface gravity of the hole, and following Page [3] we have defined $x = M\omega$. The relative magnitude of the mass and angular momentum loss rates will determine whether or not the hole will spin down to a nonzero value of a_*. To describe how the angular momentum and mass loss rates compare we define

$$h(a_*) \equiv \frac{d \ln a_*}{d \ln M} = \frac{g(a_*)}{f(a_*)} - 2. \tag{5}$$

To investigate how the mass and angular momentum evolve in time, it is convenient to define new quantities. Again following Page [3], we define

$$y \equiv -\ln a_* \ , \tag{6}$$

which we will use as a new independent variable replacing t. If the initial mass of the black hole at $t = 0$ is defined to be M_1, then a dimensionless mass variable z may be defined by

$$z \equiv -\ln(M/M_1) \ , \tag{7}$$

which has the initial value $z(t = 0) = 0$. The hole's mass, now parameterized by z, with our definitions of f and g as well as Eq. (6) and Eq. (7), will then evolve according to

$$\frac{dz}{dy} = \frac{1}{h} = \frac{f}{g - 2f} \ . \tag{8}$$

Finally we define a scale-invariant time parameter

$$\tau \equiv M_1^{-3} t \ , \tag{9}$$

with the initial value $\tau(t = 0) = 0$. The evolution of τ with respect to y is then determined by

$$\frac{d\tau}{dy} = \frac{e^{-3z}}{g - 2f} \ . \tag{10}$$

To see explicitly how a_* evolves with respect to time t we can use Eq. (10) to find

$$\frac{da_*}{dt} = -\frac{a_* h f}{M^3} \ . \tag{11}$$

At points in the evolution where $g = 2f$, or equivalently where $h = 0$, Eq. (10) is numerically bad. This is due to the fact that the hole will be described by a constant value of a_* as it continues to lose mass. For Eq. (11) we see that since the black hole is in isolation, it can only lose mass (via the Hawking process), so the function f must be positive definite. Therefore, if there is a nonzero value of $a_* = a_{*0}$, for which h is zero, then da_*/dt will be zero there and the hole will remain at that value of a_*. If dh/da_* is positive at such a point, then it represents a stable state towards which holes will asymptotically evolve. If dh/da_* is negative or zero at a point where $h = 0$, then it represents an unstable equilibrium point of a_*, and holes will evolve away from it. We will show that emission of scalar radiation may create stable states of a_* at points where $h = 0$ and $dh/da_* > 0$, depending on the mix of fields present.

The evolution of the mass and angular momentum of the hole are completely determined by Eq.(8) and Eq.(10). For collections of fields for which the black hole spins down completely, evolving towards a state described by a value of $a_* = 0$, the situation resembles that analyzed by Page. First, the initial point of the numerical integration will be taken to be nearly extremal with $a_* = 0.999$. The resulting functions $z(y)$ and $\tau(y)$ may then be used to describe evolution from other initial rotations a_{*i}, with the initial values

$$y_i \equiv -\ln(a_{*i}), \tag{12}$$

$$z_i \equiv z(y_i) = -\ln(M_i/M_1), \tag{13}$$

$$\tau_i \equiv \tau(y_i) = M_i^{-3} t_i. \tag{14}$$

A hole which began with $a_* = 0.999$ and $M = M_1$ at $t = 0$ will then have $a_* = a_{*i}$ and $M = M_i$ at $t = t_i$.

For a collection of fields for which the hole evolves to a spinning asymptotic state described by $h(a_{*0}) = 0$ (as happens, for example, with purely scalar radiation) two distinct evolutionary cases must be investigated. One of these is to start out again with a nearly

extremal hole as described above, where now the possible initial values a_{*i} range only from the initial near extremal value to the asymptotic limiting value a_{*0} for that set of fields. In the second case the initial point of the evolution will be taken to be nearly Schwarzschild with an initial value $a_* = 0.001$. In this case the value of a_* will increase as the evolution progresses, increasing towards the asymptotic value a_{*0}. The resulting evolution may be used to describe all initial states with a_{*i} ranging from the initial value up to the limiting value a_{*0} for that set of fields.

Once we have determined the evolution of the hole there are a number of other quantities we would like to investigate. The first is how the area, and hence the entropy, of the hole evolves. This can be done by calculating the area at each step of the evolution,

$$A = 8\pi M^2 \left[1 + \left(1 - a_*^2\right)^{1/2}\right] \ . \tag{15}$$

Another quantity of interest is the lifetime of a black hole that is described by an initial value of $a_* = a_{*i}$ and just evaporates today. A scale invariant lifetime θ can then be given by

$$\theta_i = e^{3z_i} \left(\tau_f - \tau_i\right) \ . \tag{16}$$

The quantity τ_f is the scale invariant time required to reach the endpoint of the evolution from the maximal initial values of the numerical integration. In Ref. [3], Page defined τ_f by $\tau_f \equiv \tau(y = \infty)$. Note that as $y \to \infty$, $a_* \to 0$. Since a black hole emitting scalar particles may not spin down to $a_* = 0$, we instead use a more general definition, namely that $\tau_f \equiv \tau(z = \infty)$. In other words, we define the lifetime in terms of the time it takes the mass to reach zero, rather than the time until $a_* = 0$ (since a_* may not ever approach zero in the general case). Here τ_i is the time it takes for a hole to evolve from an initial value of $a_* = a_{*i}$ and mass M_i to $M = 0$. Once the lifetime of a hole is known, one can then calculate the initial mass of a primordial black hole formed with $a_* = a_{*i}$ that has just evaporated within the present age of the universe t_0:

$$M_i(a_{*i}, t_0) = t_0^{1/3} \theta_i^{-1/3} = t_0^{1/3} e^{-z_i} \left(\tau_f - \tau_i\right)^{-1/3} \ . \tag{17}$$

If a primordial black hole were observed today with rotation parameter a_*, a minimum value on its initial mass could be found by letting τ_i go to its minimum value. For a black hole that evolves by spinning down, either to a nonrotating state, or towards an asymptotic state described by a_{*0}, the minimum value is simply given by setting $\tau_i = 0$, provided primordial black holes can be formed with initial values of $a_* = 1$. The other case is when the hole forms with an initial value of $a_* < a_{*0}$, so that it spins up as it evolves. In this case the limiting value of $\tau_i = 0$ again, but we must choose a limiting value for the initial value of a_*. Here we choose the initial value to be $a_* = 0.001$, which is larger than our numerical error. The minimum mass for either the spinup or spindown case is then given by

$$M_{\min}(a_*, t_0) = t_0^{1/3} e^{-z} \tau^{-1/3} \; . \tag{18}$$

III. NUMERICAL METHODS

The bulk of the numerical calculation is computing the amplification for each mode according to Eq. (3). The asymptotic forms of the radial part of the wave equation for the massless scalar field given in Eq. (2) can be used to numerically integrate Eq. (1) and obtain the function R. We followed Bardeen's method of integration described in Teukolsky and Press [4]. Initial conditions were chosen to be a purely ingoing wave at the event horizon using the asymptotic form of R as $r \to r_+$ found in Eq. (2). This solution was then integrated outward to a large value of r where it was resolved into its ingoing and outgoing components. Note from Eq. (2) that the ingoing and outgoing solutions both fall off as $1/r$ as $r \to \infty$. This creates a difficulty as both modes will contribute to the same order to the numerically integrated value of R. To separate the contributions from each mode note that a derivative of R as $r \to \infty$ gives

$$\frac{dR}{dr} \simeq -\frac{R}{r} + \frac{2i\omega Z_{\text{out}} e^{2i\omega r}}{r} \; . \tag{19}$$

Since dR/dr was already calculated in our numerical integration we can use this value and the value of R to calculate Z_{out} and then Z_{in}. The amplification due to scattering can

then be calculated from Eq.(3). This must be done for each value of l, m, and ω. This numerical integration was done using a Bulirsch-Stoer [6] method to an accuracy of one part in 10^4 for the entire integration. We found that the accuracy of the value calculated for the amplification depended on the endpoint of the radial integration. If the integration did not proceed to a large enough value of r some of the scattering would be missed. For this reason a check was made to insure that the integration had proceeded to a large enough value of r such that the value for the amplification was constant to within a part in 10^4 as well.

A final numerical integration was done to determine how the mass and angular momentum of the hole evolve in time. This was done by integrating Eq. (8) and Eq. (10). Since f and g are necessary for the integration, both were calculated at 12 values of a_* so as to match up with the data given by Page for the nonzero spin fields. We fit the values for f and g for the scalar field by a 10th order polynomial in a_*. This fitting resulted in a standard deviation for both f and g on the order of a part in 10^4. A similar fit could be obtained by using a 4th order polynomial, however the standard deviation was larger. Fitting of the nonzero spin fields was done in the manner prescribed by Page [3]. The fitting for f and g is done to save computation time by eliminating an additional integration for f and g as the integration for Eq. (8) and Eq. (10) proceeded. The integration for f and g has an overall accuracy of approximately a part in 100.

Total values for f and g may be found by summing the contributions over all of the fields

$$\begin{pmatrix} f \\ g \end{pmatrix} = \sum_s \mathrm{n}_s \begin{pmatrix} f_s \\ g_s \end{pmatrix} , \tag{20}$$

where s indicates the spin of the field and n_s is the number of fields with that spin. One can find the mass and angular momentum loss rates for an arbitrary collection of fields by using Eq. (20) together with the scalar field data provided in Table I and the results for the higher spin fields found in Table I of Ref. [3].

The final integration of Eq. (8) and Eq. (10) was done using an adaptive stepsize, fourth order, Runge-Kutta routine to an accuracy of a part in 10^4 per step. The total number of steps for a typical integration was a few hundred, resulting in an overall accuracy of

approximately a part in 100. The values of z, or equivalently M/M_1, and τ were then used to calculate the area, lifetime, the initial mass of a primordial black hole that has just evaporated, and the minimum initial mass of a primordial black hole seen today with a value of a_{*i} over the range $0 \leq a_{*i} \leq 1$.

Numerically there is a difficulty with Eq. (16) for small initial masses M_i. Note that as $\mathrm{M_i} \rightarrow 0$ the quantity $z_i \rightarrow \infty$. At the same time, the quantity $\tau_f - \tau_i$ is going to zero. It is difficult to accurately determine the evolution numerically here, and it is useful to instead use an analytic approximation. Here Page used a power series expansion to determine the lifetime when $a_* \rightarrow 0$, due to his definition of τ_f. Since in our case a_* may not tend to zero, we had to follow a different procedure. Using L'Hôpital's rule one can obtain the limiting behavior when $M \rightarrow 0$ and find that it agrees with Page's approximation of the lifetime when a_* is small [3].

$$\theta_i = \lim_{M_i \rightarrow 0} \frac{1}{3f(a_{*i})} \ . \tag{21}$$

IV. RESULTS

In 1972 Press and Teukolsky calculated the amplification of scalar waves from a Kerr black hole [7]. They noted that there was a problem with their numerical code as it was generating nonzero values for Z at $\omega = m\Omega$, which is the upper limit of the superradiant regime. In subsequent papers [4,8] they calculated the amplification for spin 1 and spin 2 fields. In both cases they note that the maximum amplification occurs for those modes which have $l = m = s$. Spin 1/2 fields exhibit no superradiance due to the fact that they are fermions and obey the Pauli exclusion principle. For all nonscalar fields, the $l = 0$ mode is a non-radiant mode. For a scalar field, this mode is radiant but does not exhibit superradiance. Sample amplification curves in the superradiant regime for a massless scalar field are shown in Fig. 1. It can be seen that all of the curves go to $Z = 0$ at $\omega = m\Omega$ as they should. To the best of our knowledge this is the first time the amplification for a scalar

field scattering off a Kerr black hole has been calculated correctly. To accomplish this we used the Bardeen transformation found in [4] which Press and Teukolsky used for the spin 1 and spin 2 fields. The amplification can be used to calculate f and g, the scale invariant mass and angular momentum loss rates respectively.

Page found that for the nonzero spin fields f is a monotonically increasing function of a_* [3]. However for the scalar field this is no longer true. As we noted in [1] and as shown in Fig. 2, for a scalar field f has a minimum located approximately at $a_* \simeq 0.574$. The existence of this minimum is due to the fact that, unlike the nonzero spin fields, the scalar field can radiate in an $l = 0$ mode and this mode dominates f at low values of a_*. The emission in this mode then decreases as a_* increases. This suggests that the $l = 0$ mode couples more strongly to the hole at low values of a_*. Emission in the superradiant modes however monotonically increase as a_* increases. These two effects combine to form the observed minimum. For both scalar and nonzero spin fields f is a positive definite function, indicating that the hole's mass always decreases into the future. This is expected since we assumed that the hole was in isolation so there would be no way for it to gain mass. Using a tenth order polynomial fit we can extrapolate our numerical values for f to a $a_* = 0$ and compare with known results for Schwarzschild black holes. We find that $f \to 7.44 \times 10^{-5}$ as $a_* \to 0$, which agrees to three significant digits with the previous results found by Simkins [9] and Elster [10].

For the scale invariant angular momentum loss rate, Page found for nonzero spin fields that g is a monotonically increasing function of a_* [3]. As shown in Fig. 3 this is also true for the scalar field. The reason the scalar field result is qualitatively similar to the nonzero spin fields is that the $l = 0$ mode carries off no angular momentum so it makes no contribution to g. The form of g can then be understood purely in terms of superradiance, which causes g to increase as a_* increases. A table of values for f and g for a massless scalar field can be found in Table I which complement those given for the nonscalar fields by Page [3].

Using our results for f and g we can calculate h from Eq. (5). Note from Eq. (11) that da_*/dt will be zero only when $h = 0$, since f is positive for all a_* for a hole in isolation.

Fig. 4 shows the behavior of h due solely to particle emission by a scalar field. The most important feature is that $h(a_*) = 0$ at a value of $a_* \simeq 0.555$ as seen in the figure and noted in [1]. A black hole that forms with a value of $a_* > 0.555$ will have $h(a_*) > 0$ so by Eq. (11), da_*/dt will be negative and the value of a_* will decrease as the hole evaporates, tending towards $a_* = 0.555$. In contrast a hole that forms with $a_* < 0.555$ will have $h(a_*) < 0$ so da_*/dt will be positive and the value of a_* will increase towards $a_* = 0.555$. Thus, when only emission of scalar field particles is considered, a black hole with any nonzero initial angular momentum will evolve towards a state with $a_* \simeq 0.555$. This should be compared to what was found for nonzero spin fields where the hole rapidly evolves to a state characterized by $a_* = 0$.

Page found that the value of h at $a_* = 0$ satisfied an approximate linear relationship with the spin for the nonzero spin fields he examined:

$$h_s(a_* = 0) \simeq 13.4464s - 1.1948 \; . \tag{22}$$

Extrapolating this to $s = 0$ indicates that $h_0(a_* = 0)$ is negative, which led Page to suggest that emission of scalar particles might prevent a black hole from spinning down. While this conclusion is confirmed by our calculations, we find $h_0(a_* = 0) = -0.806$, which does not satisfy Eq.(22), indicating that the approximate linear relation breaks down for the scalar case.

As seen in Fig. 5, h is a positive definite function for the nonzero spin fields. Since we have found for a scalar field that h has a region in which it takes negative values it seems clear that whether $h = 0$ at some nonzero value of a_* depends on the collection of fields present in nature or considered in a model problem.

We can now numerically evaluate Eq. (8) and Eq. (10) and determine how a rotating hole's mass and angular momentum will evolve with solely a scalar field and with a mixture of fields. For simplicity, we will ignore emission by massive fields. We will consider cases including massless neutrinos, and also cases without neutrino contributions. We choose sets of species that correspond to sets chosen by Page [3] to facilitate comparison. Within the

standard model the only fundamental scalars are the Higgs bosons, which have a relatively high mass. By the time the temperature is high enough to emit these massive scalar particles, many other higher spin particles will be produced in substantial numbers and the overall situation will become overly complicated due to possible symmetry restoration. For this reason the scalar fields will be taken to be truly massless in our discussions.

The first case we will consider will be that of a black hole emitting radiation from a single scalar field as it evaporates. In such a case, the hole will evolve towards a state specified by $a_{*0} \simeq 0.555$. As seen in Fig. 6, a hole that starts out nearly extreme with a value of $a_* = 0.999$ has reached a state characterized by a_{*0} by the time it has lost 85 % of its initial mass. As $a_* \to a_{*0}$, we find that $a_* - a_{*0}$ is proportional to the mass of the hole squared.

In contrast a hole that starts out nearly nonrotating, with $a_* = 0.001$, stays at roughly the same value of a_* until it has only approximately 1 % of its initial mass. At that point its value of a_* increases and reaches $a_{*0} \simeq 0.555$ once it has about 0.01 % of its initial mass remaining. This evolution is shown in Fig. 7. As $a_* \to a_{*0}$, we again find that $a_* - a_{*0}$ depends quadratically on the mass.

The second case we consider is one in which the hole is emitting particles from a single massless scalar field and, in addition, from the known massless higher spin fields. Specifically we will allow the hole to emit particles from a single spin 1 and spin 2 field in addition to a single scalar field. This changes the evolution only slightly from that found by Page; in particular, the hole still appears to spin down completely.

The third case we describe involves massless spin 1 and spin 2 fields and in addition three spin 1/2 fields representing the neutrino flavors. This set will be called the canonical set of fields. In addition to the canonical set we also allow a single massless scalar field to be present. The resulting evolution is very similar to that found by Page for the canonical set of fields; the hole again rapidly spins down to a Schwarzschild state. The three additional neutrino fields have a very small effect on the evolution, as these fields are not superradiant.

Since the addition of a single scalar field is seen to have little effect when the multiple known fields of nature are included in the mix, we next determined how many scalar fields

would be necessary in addition to the canonical set of fields to allow the hole to evolve towards a state described by nonzero a_*. Given the canonical set of fields, we find it takes a minimum of 32 massless scalar fields to cause the hole to evolve to a state with a_* nonzero. This combination of fields will evolve to a state described by $a_* = 0.087$. This indicates that if there were at least 32 massless scalar fields in nature, then evaporating black holes could evolve towards an asymptotic state described by a nonzero value of a_*. It is unlikely this will be realized, since there are at present no known massless scalar fields in nature.

In Fig. 8 we see the scale invariant lifetime given by Eq. (16) for a primordial black hole that was formed with rotation parameter a_{*i}, which has just disappeared and which evaporated by purely massless radiation into a single species.

In Fig. 9 the fractional mass is plotted versus the fractional time for holes that are emitting purely in one species. The emission due to a single scalar field allows the hole to lose mass more slowly than in those cases which emit nonzero spin particles. Here the nonzero spin fields each carry off more energy than a single scalar field.

In Fig. 10 the rotation parameter is plotted versus the fractional lifetime for holes that are emitting purely in a single species. As before we see that purely scalar emission causes the hole to approach an asymptotic value of $a_* \simeq 0.555$, either from above or below, depending on the initial state of the black hole.

In Fig. 11 the initial mass of a black hole that has just disappeared today via radiation from a single massless field is plotted against the initial value of its rotation parameter a_{*i}. As Page did, we assume the age of the universe is $t_0 = 16 \times 10^9$ years which implies in our units that $t_0^{1/3} = 4.59 \times 10^{15}$ g. We used the limiting form of the lifetime shown in Eq. (21) to calculate the lifetime as $M_{\mathrm{i}} \to 0$.

In Fig. 12 the minimum mass that a primordial hole could have had at the time of its formation is plotted against the value of the rotation parameter a_* it has today. Here the two scalar curves representing the differing spinup and spindown evolutions show significantly different behavior, from each other and from the higher spin fields. In particular, note that if only massless scalar emission occurred, a hole observed with a_* slightly greater than 0.555

would have a significant uncertainty in its initial mass, due to the steepness of the spin 0 curve shown here.

In Fig. 13 the ratio of the horizon area to the initial area is plotted versus a_* for each of the individual fields. Essentially, a_* is being used here as an alternate time parameterization to spread out the rapid evolution which occurs near $a_* = 1$. Note that the area actually increases initially as a_* decreases for nearly extreme holes. The evolution of the area can be described by differentiating Eq. (15)

$$\frac{dA}{dt} = \frac{A}{M^3}\left(\frac{hf}{(1-a_*^2)^{1/2}} - g\right) . \tag{23}$$

Since h and f are both positive as $a_* \to 1$ the first term on the right hand side of Eq.(23) dominates, causing the initial increase in the area. Since the entropy is one quarter the horizon area these results also describe the evolution of the black hole's entropy.

V. DISCUSSION

In this paper we have described how the emission of massless scalar field radiation via the Hawking process affects the evolution of a Kerr black hole in isolation. We find that for low values of a_* the hole's dominant emission mode for the scalar field is the $l = 0$ mode. This follows the same trend as the nonzero spin fields, namely that the $l = m = s$ mode dominates the emission. We found that the scale invariant mass loss rate f has a minimum located at a value of $a_* \simeq 0.574$ for the massless scalar field while in contrast Page found that for the nonzero spin fields f is monotonically increasing with a_*. This is due to the fact that the $l = 0$ mode is not a radiant mode for the nonzero spin fields and this mode dominates the emitted scalar radiation when a_* is small. As a_* increases the higher l modes, with $m \neq 0$, which are superradiant, begin to increase in strength while the $l = 0$ mode emission decreases due to poor coupling with the hole at high values of a_*. Together these two effects combine to form the minimum. We also found that the scale invariant angular momentum loss rate for the scalar field is qualitatively similar to the results Page found for the nonzero

spin fields, being a monotonically increasing function of a_*. Combining these two results we showed that the quantity h which describes the rate at which the angular momentum and mass of the hole are changing relative to each other reaches a value of zero at $a_* \simeq 0.555$ for a hole emitting purely massless scalar radiation. This implies that, for the evolution of a black hole-quantized scalar field system, a black hole which possesses any nonzero initial angular momentum will evolve towards an asymptotic Kerr black hole state described by this value of a_*, which we argue is a stable point of the evolution. By comparing our value of $h(a_* = 0)$ with that predicted by Page's hypothesized linear relationship for the nonzero spin fields we have shown that the proposed relationship breaks down for the scalar case.

Using our results for f and g for the scalar field along with the values of f and g given by Page for the nonzero spin fields, one can calculate the evolution of a Kerr black hole evolving by the emission of a collection of particles of arbitrary spin. We have calculated the evolution of a Kerr black hole for a number of cases in which the hole is allowed to emit radiation from different fields. For a hole that is emitting only radiation from a single massless scalar field in addition to radiation from the known massless fields (electromagnetic and gravitational), the evolution follows much the same trend as that found by Page for purely nonzero spin fields. We also find that the neutrino fields have very little effect on the evolution due to the fact that they are not superradiant. If we allow the hole to emit radiation from the two known massless nonzero spin fields and three neutrino fields we find that the hole will asymptotically approach a nonzero value of a_* only if there are 32 or more massless scalar fields present.

We also found that the scale invariant lifetime is decreased when a hole is allowed to radiate massless scalar particles in addition to the canonical set due to the addition of new radiative channel. The initial mass of a black hole that has just now evaporated correspondingly is thus increased, although only slightly. We also found that by allowing the hole to emit into a massless scalar field in addition to the canonical set of fields slowed the rate at which angular momentum is lost by the hole. The minimum mass for a hole that is seen today with rotation parameter a_{*i} deviates in a very insignificant matter whether or not the

hole is allowed to radiate into the canonical or canonical plus scalar set of fields. Finally the area, and consequently the entropy, was found to be lower at each value of a_{*i} for emission into the canonical plus scalar set of fields versus just the canonical set of fields, again due to the addition of another radiative channel in the form of the scalar field.

ACKNOWLEDGMENTS

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FIGURES

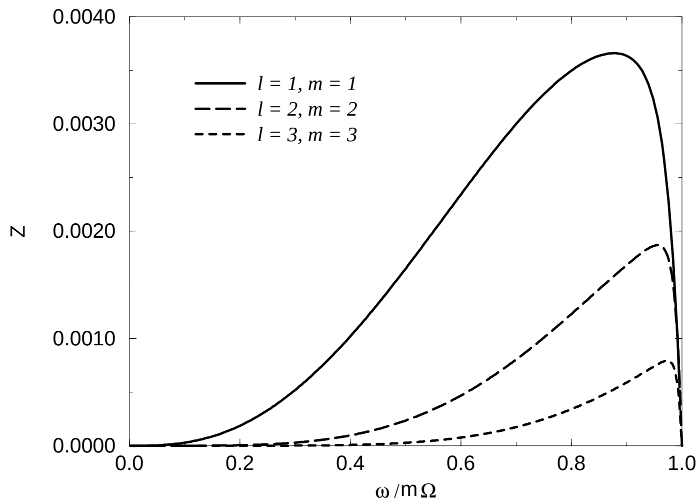

FIG. 1. The amplification Z of massless scalar radiation from a Kerr black hole with rotation parameter $a_* = 0.99$, plotted in the superradiant regime, $0 \leq \omega \leq m\Omega$. Note the $l = 0$ mode is not superradiant and is therefore not shown.

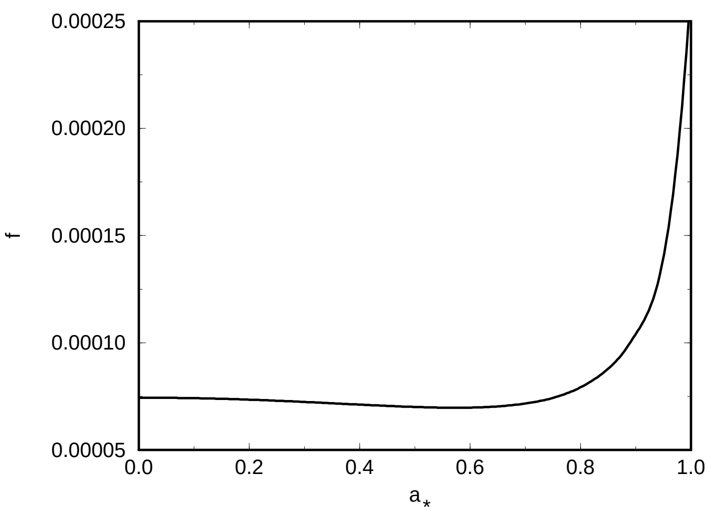

FIG. 2. The scale invariant mass loss rate is shown versus the rotation parameter for a single massless scalar field. There is a minimum located at $a_* = 0.574$. In contrast, for nonzero spin fields the function f is monotonically increasing.

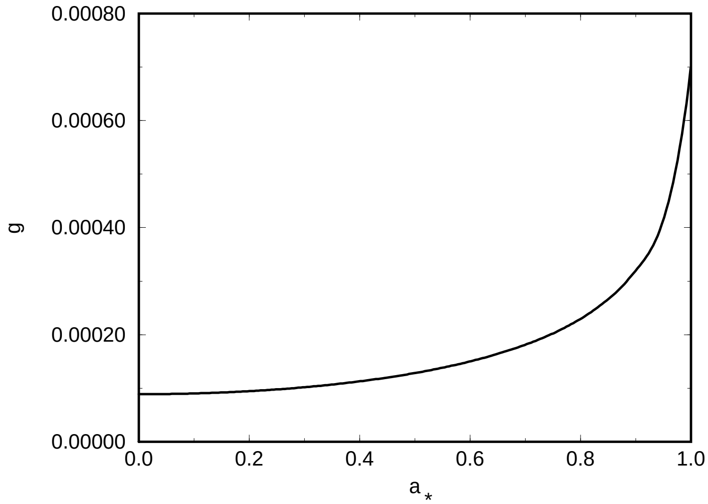

FIG. 3. The scale invariant angular momentum loss rate is shown versus the rotation parameter a_* for a single massless scalar field. The curve is qualitatively similar to those for the nonzero spin fields, being a monotonically increasing function of a_*.

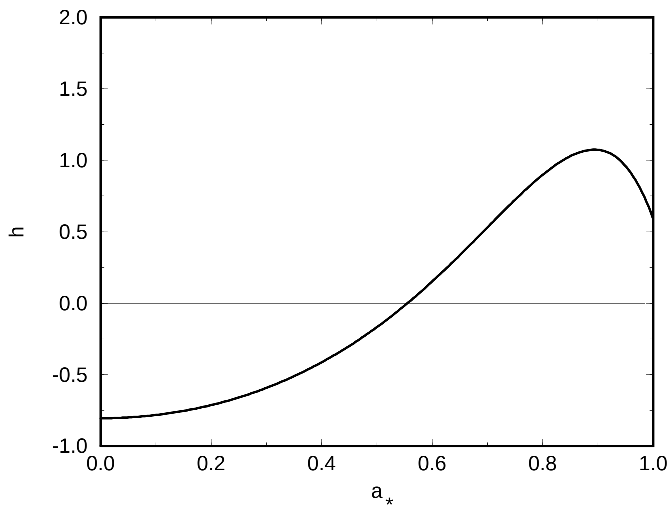

FIG. 4. The scale invariant quantity h which describes the rate of change of the angular momentum relative to that for the mass is plotted versus the rotation parameter for a single massless scalar field. The function $h(a_*)$ has a zero at $a_* \simeq 0.555$. A hole that forms with $a_* > 0.555$ will then spin down to that value as it evolves while one that forms with $a_* < 0.555$ will spin up towards that value.

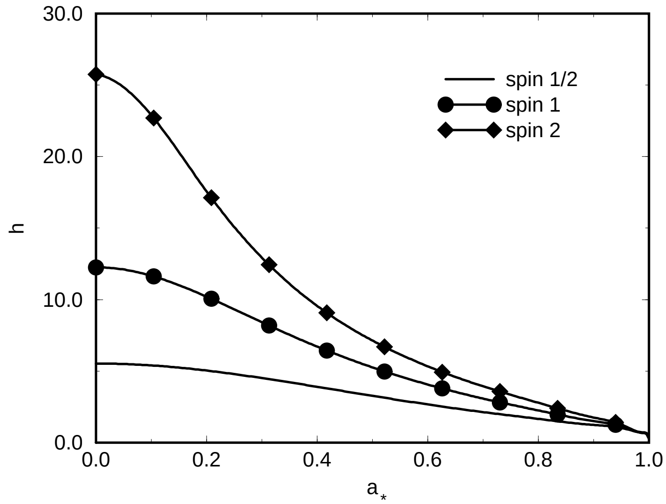

FIG. 5. The scale invariant quantity h is plotted against the rotation parameter for each of the nonzero spin fields. The function for each of the fields is positive definite for all values of a_*, showing that a Kerr black hole emitting radiation from nonscalar fields loses angular momentum faster than mass.

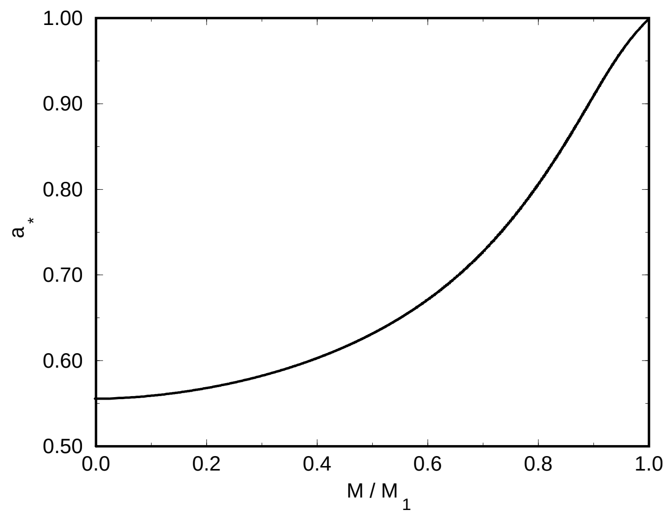

FIG. 6. The mass of a black hole evaporating solely by emission of radiation from a single massless scalar field is shown plotted versus its rotation parameter a_* for an initially rapidly rotating hole. The black hole evolves to a state characterized by $a_* \simeq 0.555$ from an initial state characterized by $a_* = 0.999$ and initial mass M_1. The evolution of a black hole that has a different initial value of $a_* = a_{*i}$, but in the range $0.555 < a_{*i} \leq 1$ can be found by locating the desired value of a_{*i} on the curve and rescaling the horizontal axis so that $M/M_1 = 1$ at that point.

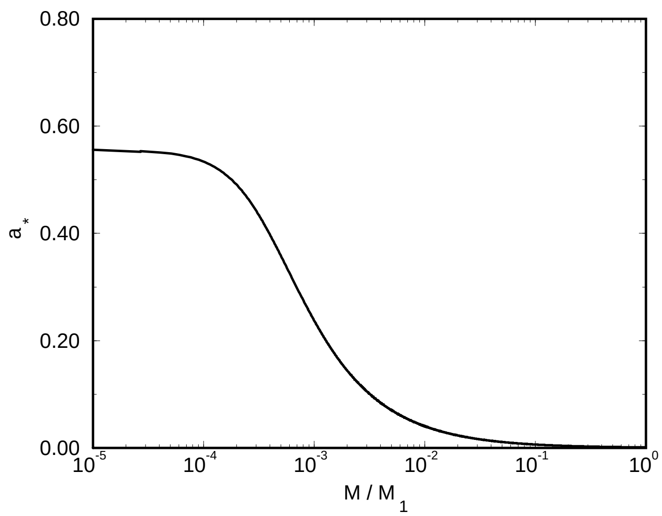

FIG. 7. The mass of a black hole evaporating solely by emission of radiation from a single massless scalar field is shown plotted versus its rotation parameter a_* for an initially slowly rotating hole. The black hole evolves to a state characterized by $a_* = 0.555$ from its initial state characterized by $a_* = 0.001$. The evolution of a black hole that has a different initial value of $a_* = a_{*i}$, but in the range $0 \leq a_{*i} < 0.555$ can be found by locating the desired value of a_{*i} on the curve and rescaling the horizontal axis so that $M/M_1 = 1$ at that point.

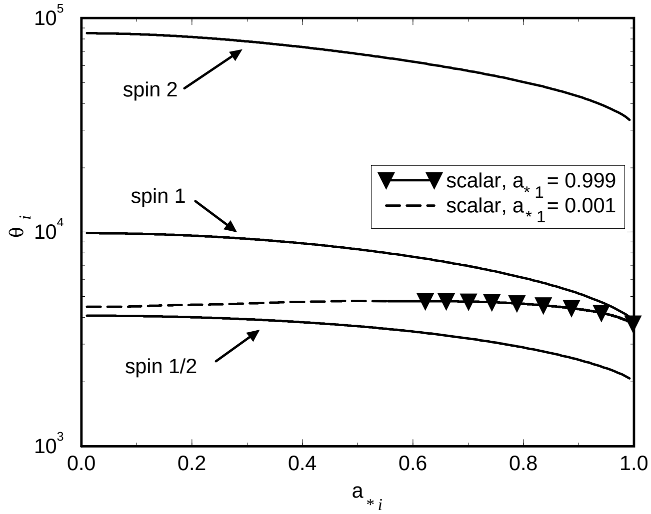

FIG. 8. The scale invariant lifetimes for primordial black holes emitting radiation from differing single massless fields is plotted versus the initial value of the rotation parameter, a_{*i}, that the black hole formed with. Here we see the two distinct scalar evolutions, one which spins up from a nearly Schwarzschild state and one which spins down from a nearly extreme state, both asymptotically approach $a_* \simeq 0.555$. In this and the following figures, the two different evolutionary paths due to emission of purely scalar particles from the black hole are differentiated by the use of a symbol on one of the curves. The number of symbols is not related to the number of data points and their use is intended only to help clarify the figure.

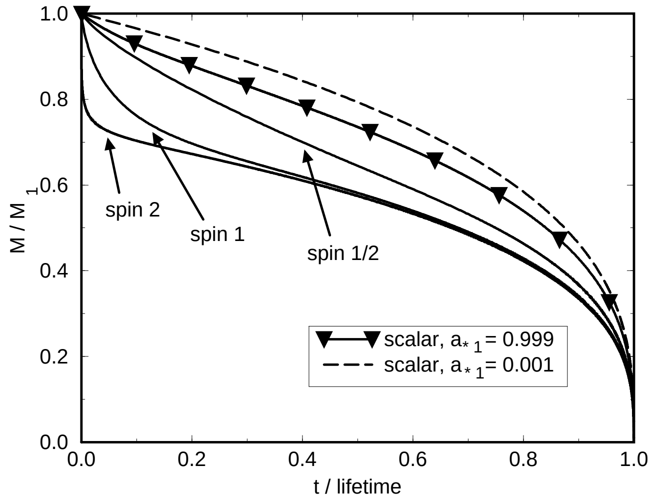

FIG. 9. The fractional mass is plotted against the fractional lifetime for a black hole that is emitting radiation from a single species. Pure massless scalar radiation decreases the mass loss rate relative to the nonzero spin fields. The two different scalar curves represent a hole that is starting in a nearly extremal state or a nearly Schwarzschild state.

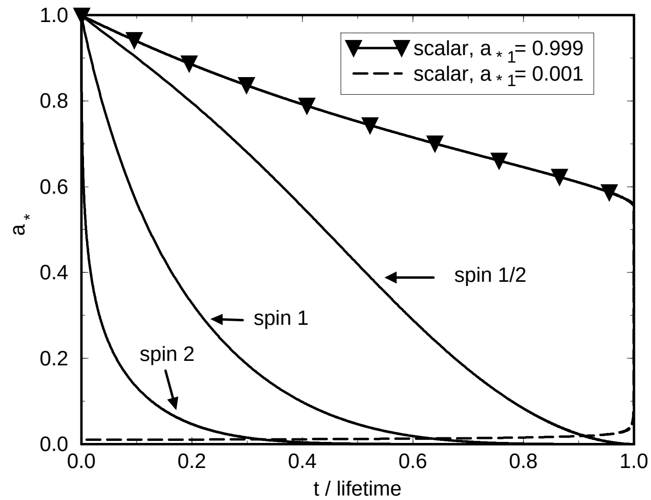

FIG. 10. The rotation parameter is plotted versus the fractional time for a black hole that is evaporating by radiation from a single field. Massless scalar radiation causes the hole to spin down more slowly than in the nonzero spin cases. The evolution of a black hole that starts out with a different value of $a_* = a_{*i}$ can be found by shrinking the vertical axis from the top for those holes starting at a nearly extremal state to the desired value of a_{*i} and rescaling the horizontal axis so that $t_{\text{initial}} = 0$. The same can be done for the hole which begins nearly Schwarzschild and is radiating into a single massless scalar field by the above process, but starting from the lower left corner.

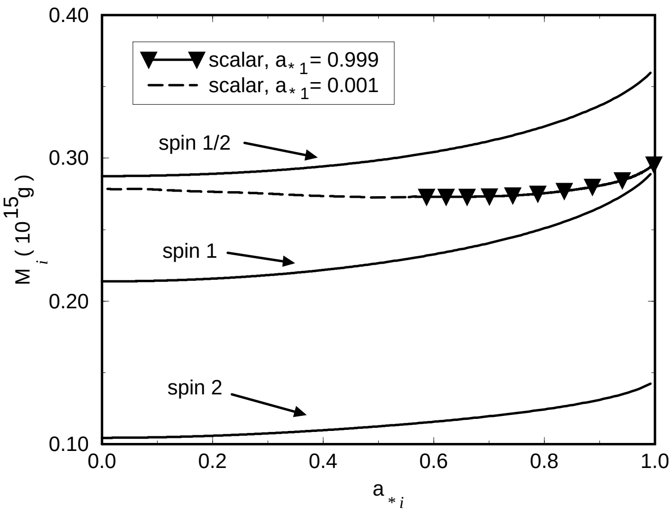

FIG. 11. The initial mass of a primordial black hole that has just disappeared within the present age of the universe by emission of radiation from a single massless field is plotted versus the value of the rotation parameter when it formed, a_{*i}. Here the two distinct evolutions by emission of purely scalar particles, one spinning up and one spinning down, can be seen to converge on a state described by $a_* \simeq 0.555$.

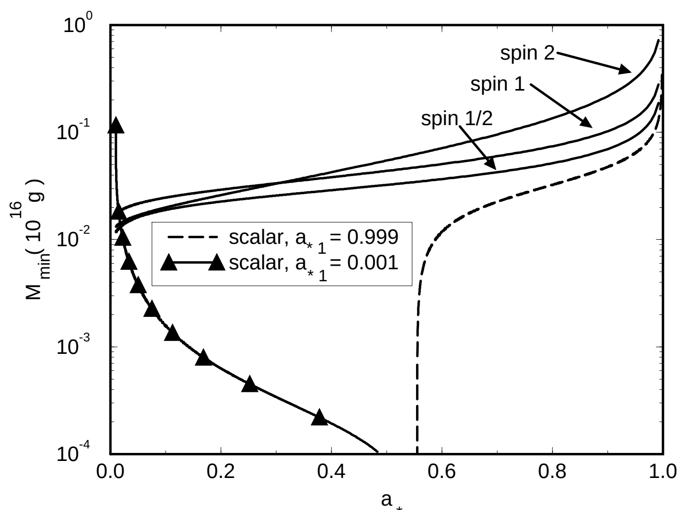

FIG. 12. The minimum mass of a primordial black hole that has been seen today is plotted against the value of the rotation parameter that it has today, a_{*i} for black holes emitting radiation from single massless fields . For a hole that is emitting purely scalar particles, both the initially near extremal and near Schwarzshild evolution curves asymptotically approach a state characterized by $a_* \simeq 0.555$.

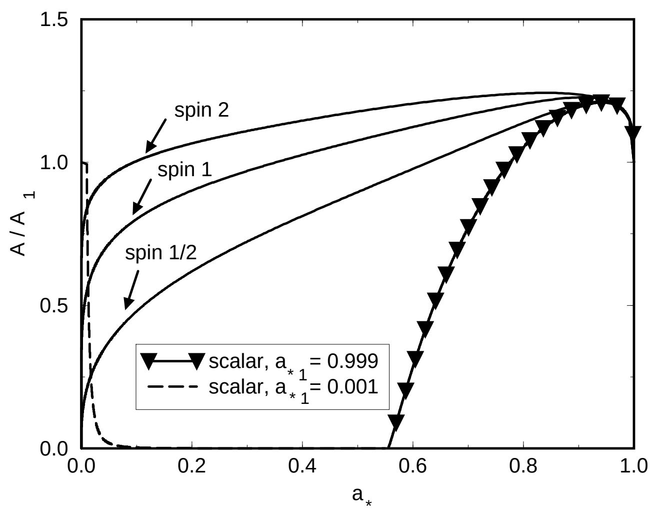

FIG. 13. The fractional area is plotted versus the rotation parameter for a black hole which is emitting radiation from a single field. Pure scalar radiation causes the area to decrease more rapidly (as a function of a_*) than for the nonzero spin fields, particularly for initially slowly rotating holes. The evolution of the area from a black hole that starts out with a value of $a_{*i} > 0.555$ can found by shrinking along the horizontal axis from the right hand curve to insure that $A/A_1 = 1$. For a hole that is emitting only scalar particles and which forms with $a_{*i} < 0.555$ the same process can be followed, only shrinking from the left curve rather than the right. This set of curves also represents the evolution of the entropy.

TABLES

TABLE I. The quantities of f and g for a single massless scalar field are shown for 12 values of a_*. The values of a_* were chosen to match those used by Page for the nonzero spin fields.

a_*	f	g
0.01000	7.429×10^{-5}	8.867×10^{-5}
0.10000	7.442×10^{-5}	9.085×10^{-5}
0.20000	7.319×10^{-5}	9.391×10^{-5}
0.30000	7.265×10^{-5}	1.024×10^{-4}
0.40000	7.097×10^{-5}	1.125×10^{-4}
0.50000	6.996×10^{-5}	1.281×10^{-4}
0.60000	7.008×10^{-5}	1.507×10^{-4}
0.70000	7.119×10^{-5}	1.803×10^{-4}
0.80000	7.969×10^{-5}	2.306×10^{-4}
0.90000	1.024×10^{-4}	3.166×10^{-4}
0.96000	1.551×10^{-4}	4.515×10^{-4}
0.99000	2.283×10^{-4}	6.160×10^{-4}
0.99900	2.625×10^{-4}	6.905×10^{-4}
0.99999	2.667×10^{-4}	6.997×10^{-4}